PROGRAM FOR SOLVING PROBLEMS AS METHOD FOR DEVELOPMENT OF LOGIC THINKING IN SCHOOL CHILDREN

PROGRAMA PARA RESOLVER PROBLEMAS COMO MÉTODO PARA O DESENVOLVIMENTO DO PENSAMENTO LÓGICO EM ESCOLARES

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ABSTRACT

The goal of the present work is to show the method for teaching the process of solving problems in primary school. The program created on such purpose, consider logical structure of the process of solution and the content of orientation. Participants of the study were regular pupils of private primary school from the city of Puebla, Mexico. The children were tested before and after participation in the program. Qualitative assessment included the tasks for logical relations and problems solution. The program was applied during 30 sessions in classroom. The teaching process was modeled as joined activity between teachers and pupils. The results showed that the program was useful for positive development of mathematical and logical thinking. After participation in the program, the pupils could solve the problems and relate mathematics data to the verbal text of the problems, which had not been observed during the initial assessment. As conclusions, we argue for the necessity to create exact and specific guided orientation to construct mathematics sessions in classrooms in primary school. At the level of modern development of historical and cultural approach, the concept of the zone of proximal development might be related to orientation as essential part of intellectual activity.

Key words: problem solution, logic thinking, teaching methods, school age, activity theory

RESUMO

O objetivo do presente trabalho é mostrar o método para ensinar o processo de resolução de problemas na escola primária. O programa criado com esse objetivo, considera a estrutura lógica do processo de solução e o conteúdo da orientação. Os participantes do estudo eram alunos regulares do ensino fundamental privado da cidade de Puebla, no México. As crianças foram testadas antes e depois da participação no programa. A avaliação qualitativa incluiu as tarefas para as relações lógicas e resolução de problemas. O programa foi aplicado durante 30 sessões em sala de aula. O processo de ensino foi modelado como atividade conjunta entre professores e alunos. Os resultados mostraram que o programa foi útil para o desenvolvimento positivo do pensamento matemático e lógico. Após a participação no programa, os alunos chegaram a resolver os problemas e relacionar os dados matemáticos com o texto verbal dos problemas, que não foi observado durante a avaliação inicial. Como conclusões nós argumentamos a necessidade de criação de orientação exata e específica para a construção de sessões de matemática na sala de aula na escola primária. Ao nível do desenvolvimento moderno da abordagem histórica e cultural, o conceito de zona de desenvolvimento proximal pode ser relacionada à orientação como parte essencial da atividade intelectual.

Palavras-chave: solução de problemas, pensamento lógico, métodos de ensino, idade escolar, teoria da atividade.

1. Introduction

The process of logic thinking requires of analysis and synthesis of information, which might be presented as sensory images of objects of life and as mental images of verbal concepts (Galperin, 2009a). According to the activity theory, the process of thinking is not an isolated function, but represents an objectal activity. Such activity can take place on different levels: material objects, materialized symbols, perceptive images of objects, generalized complex symbolic symbols, external and internal verbal actions (Solovieva, 2014). Only this last level, the level of interval verbal actions, is normally understood as a thinking process. However, in case of intellectual development of children, formation of thinking as internal verbal process starts by the objective realization of intellectual actions in external level with objects and concrete images (Piaget, 1953; Vigotsky, 1991; Leontiev, 1983; Obujova, 1997). Within the activity theory, such process of formation of intellectual actions from external up to internal level is not a spontaneous, but a guided process. Within historical and cultural conception and the activity theory, it was also shown that intellectual actions might include logic relations of cause and consequence, spatial and temporal relations starting from material stage (Galperin, 2002, 2009b; Talizina, 2001). Language takes important part in the process of intellectual actions; at the same time, intellectual actions might not be reduced to language as an isolated function. Language is essential for reflection and generalization of logic relations of cause and consequence, of temporal and spatial relations between objects. The acquisition of such relations and transition of intellectual actions from material level to the level of internal verbal actions is a gradual process, which takes a lot of time and starts at school age (Rubinstein, 1963; Luria, 1985). In case of mathematic problems, specific signs and symbols together with abstract formulas and schema are used. Dominium of such means of cognition represents very complex and long process, which might not be understood without consideration of participation of social institutions (Tomasello, 1999).

Piaget's theory of intellectual development has established that logic operations appear spontaneously as a manifestation of stages of maturation of human nervous system, which is culminated on the level of formal verbal operations (Piaget, 1973). According to this author, only at this stage, real intellect is formed with reflexive understanding of logic relations of cause and effect, space and time relations between objects and situations. Representatives of the activity theory have expressed the opposite point of view. Zaporozhets (1986) has shown that children of the age of 5 and 6 years are capable to achieve logic reflection in intellectual material actions during oriented collaboration with adults. Specific orientation in the whole situation, in temporal, causal and temporal relations between objects prior to the solution of the problem was always required and provided by an adult. Luria (1985) has shown that in complex experiments with constructions in groups of children of pre-school age, logic orientation in details has always to pass from level of graphic representation and orientation to the level of verbal logic conclusions.

Salmina y Filomonova (2010) achieved different studies with children of 5, 6 and 7 years old. Their results show that children can develop abilities before complex mathematical thinking such as logic, symbolic and numeric operations. Actions of seriation, classification and conservation are related to logic operations; actions of codification and decodification by multiple choices are related to symbolic operations; reciprocal correspondence, usage of units for measuring and comprehension of the difference between quantity and measure, ordinate and coordinate counting are related to numeric operations. All three components (logic, symbolic and numeric) are absolutely necessary for formation of mathematical abilities and concept at primary school and for future levels of education (Salmina, 2001). Different authors have developed methods for introduction of actions with these components at preschool and school age (Salmina, 2001; Talizina, 2001; Salmina & Filimonova, 2010; Rosas & Cols., 2013; Solovieva & Quintanar, 2016). Such actions can be formed in joint activity guided by adults on material and graphic level.

Different psychological studies have shown that these three are not sufficiently developed at preschool and first grades of primary school. Solovieva et al. (2013) have assessed the level of formation of these components in Mexican preschool children between 5 and 6 years old in different social groups: private pre-school institutions, official urban and rural pre-school institutions. Some of the proposed tasks were one to one correspondence, seriation and comparison of empiric concepts. The results have detected the children of all three social groups have managed to fulfill the tasks only after orientation provided by an adult. That means that such tasks are accessible only in situation of collaboration and might be situated within the zone of the proximal development. That is a positive moment. The negative moment is that the methods used traditionally at schools do not suppose step-by-step orientation and presentation of all components of intellectual tasks while introduction of arithmetical knowledge at school. Teachers do not use any orientation at all and the majority of school tasks consist of repetition, coping and memorization without reflection of logic situations.

Another study (Zárraga et al. 2012) shows an Image of the organization of the content and orientation for introduction of mathematical abilities in a group of suburban preschool children. Solovieva, Ortiz & Quintanar 2010 show the experience of organization of the content and orientation suitable for teaching of concept of number in indigenous children who speak Nahuatl and Spanish. Actions of addition, subtraction, multiplication and division were also formed in this study.

Such results show that both logic and symbolic abilities might be formed at preschool age only as a result of joint external orientated activity. The object of such activity always required logic or symbolic operations. The process of formation is at the same time the process of reflexive intellectual activity, which starts from external and passes later up to internal verbal conceptual level.

The previous studies have shown that the process of problem solution has to include specific orientation in order to understand structural components of the problem: final and complementary questions, numeric data, identification of steps for realization of operations and final verification (Luria, 1985; Obujova, 1977; Tsvetkova, 1999; Rosales, Orrantia, Vicente & Chamoso, 2008; Talizina, 2009).

From the point of view of a reflexive organization of a problem-solving activity, the actions of the pupil have to start by identifying the steps of the solution (Nicola & Talizina, 2001). For doing so, the pupils have to identify the final question of the problem as an essential part of the whole structure. All following steps depend on the final question. Later on, the pupils have to achieve an analysis of conditions, under which the question is situated. The conditions of the problem always describe a kind of concrete situation, in which some numeric (mathematic) data is inserted. The teacher is obliged to provide orientation for analysis and synthesis of all data of the problem in relation to the final question. Specific intellectual actions have to be considered, provided and used together with the order of the operations (Talizina, 2009). On the basis of the following findings of activity theory, we supposed that intellectual actions required for the process of problems solving are much broader than the knowledge of mathematical relations (operations) alone. We supposed that to understand the text and all the words used in the problems is important also during the process.

The goal of the present article is to show an Image of teaching method for problem solution as a part of mathematical knowledge. The methodology considers all theoretical and methodological positions of the activity theory mentioned above. Such positions may be summarized as follows: 1) analyses of the content of the action of problem solution; 2) design and usage of the necessary guidance for problem solution; 3) joint intellectual action including all children of the classroom and teacher; 4) logic separation of verbal and numerical data of the problems. The teaching method was worked out as a program, which was applied to Mexican children of the second grade of primary school.

2. Methodology

2.1 Participants

Pupils of the second grade of primary school of the city of Puebla (Mexico) participated in the study. They were all regular pupils and belonged to a private school zone. The school had just been inaugurated and the classroom consisted only of 4 children: three boys and one girl. The average age was 7.5 years. The program was applied after previous formation of numerical concepts within same group of participants. Participants have also acquired previously actions of addition, subtraction, multiplication and division (Rosas, Solovieva & Quintanar, 2014; Rosas, Solovieva, García & Quintanar, 2013).

2.2 Procedure

All children were assessed individually before starting with the program of problem solution and after finishing their participation in the program.

The process of study based on gradual formation, which was called in the literature as genetic casual method, was proposed by L.S. Vygotky and developed within the activity theory (Leontiev, 2003). Formative study consists in the organization of the process of active interaction between children and teacher; we might say that it is an interaction between experimenter and participant. The teacher (experimenter) has to know perfectly all features of the process of interaction and to provide appropriate orientation in all steps. Joint intellectual actions are included with consideration of analyses of all essential elements of the problem-solution content (Talizina, 2000; Nikola y Talizina, 2001; Solovieva, 2013).

Qualitative analysis of the process and of the data of assessment before and after the participation in the program took place afterwards.

2.3 Instruments

The assessment included specific tasks directed to determine the level of conceptual dominion and reflection on the process of solution of mathematical problems. The tasks were based on the content of the Protocol for verification of School Success in Primary School (Solovieva & Quintanar, 2012) and include assessment of writing, reading and mathematics abilities. The content of the tasks

a) What is longer: 3 cm or 1 m?

b) What is heavier, 5 liters or 2 kilograms?

c) What period of time is longer, two quarters of hour or a half hour?

d) There were 7 birds on the tree, 3 of them flew away. How many birds remained on the tree?

e) There were 2 birds on the tree, 4 more birds arrived. How many birds are there now?

f) 2 birds left and 3 birds remained. How many birds were there at the beginning?

g) If the price of one toy is 7 pesos and Gerardo wants to buy 4 toys, how many pesos Gerardo must pay?

2.4 Program for formation of problem solution

The goal of the formation program was to offer and to establish the content and structure of orientation for the process of problem solution. Specific external means as symbolic formulas were included in the content of orientation. The actions of the program were fulfilled firstly on perceptive and later on external verbal level; firstly in-group and secondly individually by each pupil according to psychological conception of exteriorization (Vigotsky, 1996; Galperin, 2009b). The program was fulfilled in 30 sessions in classroom during 1 hour and within regular timetable of the college. Regular teacher of primary school and experimenter were the adults who took part in the process together with four children.

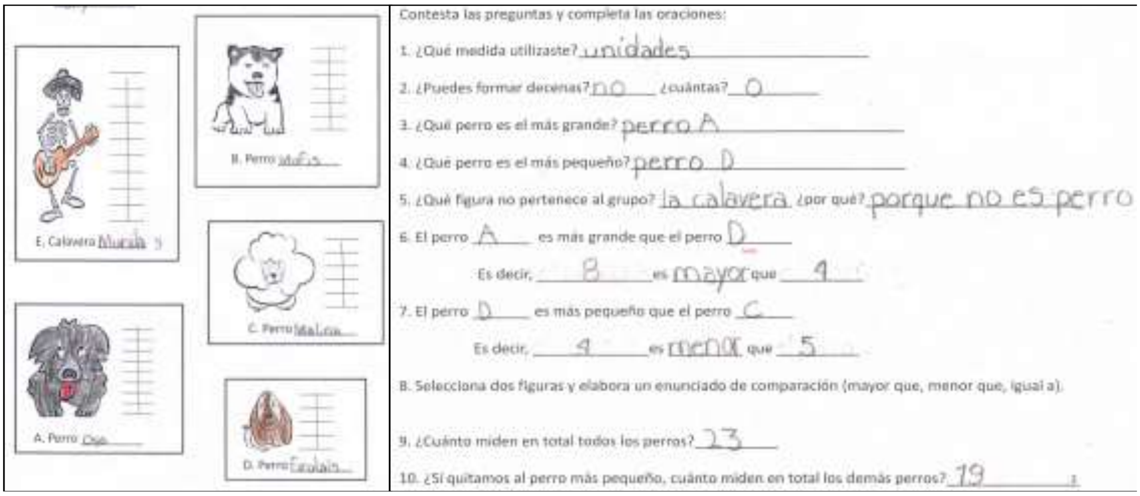
The program included the work in stages: orientation with numeric logic relations without problems, solving of simple problems, solving of complex problems, invention of problems with external orientation and independent creation and solution of the problems.
The total of 324 arithmetic problems were solved. The problems were divided into two groups: simple and complex. Simple problems consisted in one operation and included direct relation between verbal data and operation. Complex problems consisted in two operations and included indirect relations between verbal data and operation (Tsvetkova, 1999).

2.5.Content of the program for problems solution

The program at all stages assumed joint participation of teachers and pupils and included aspects of orientation and problems provided by teacher and problems created by the pupils as the process and the result of the work with the program. The part of orientation included the content of orientation, usage of external means of orientation, questions used as a part of orientation, elaboration of orientation cards by pupils.

As the content of orientation, before starting with problem solutions, the pupils have fulfilled the exercises for analysis of the data of the problems. That means that the pupils learned how to understand the problem before starting with the real solution. Analysis of the content of the problems included the verbal part (reading) and representation of the data with the help of signs (image 1). Exercise 1 shows how the pupils had to complete the questions related to the verbal context of the problems.

The goal of this exercise was to answer a variety of questions focused on the identification of the measurement and the semantic group, the comparison between objects (lesser than, greater than or equal), and the numeric representation of these comparisons. The children used mathematical signs “less than” (<), “greater than” (>) and “equal” (=) to understand the relationships between the magnitudes worked. Specific questions for identification of units and relations between the objects were used as is shown in Image 1.

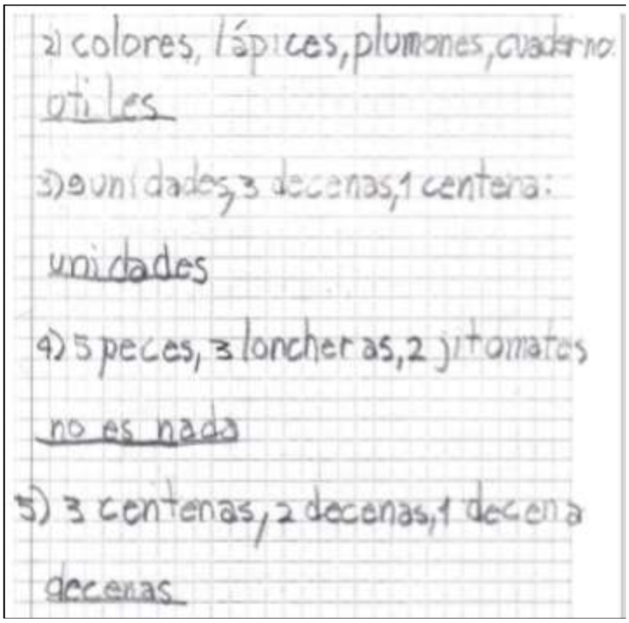


Answer the questions and complete the sentences about the length of the birds on the picture:

1. What measure did you use?__units____
2. Can you form units of tens? _no_ How many do you have?___0_
3. Which dog is the biggest?___the dog A_____
4. Which dog is the smallest of all? the dog D_____
5. Which image is outside the group?__
6. The dog __A___is bigger than the dog __D__ This means, _8 units__are more than ___4 units_(when we use the same unit for measuring).
7. Dog __D_______is smaller than __C___ This means, _that 4 units__are __smaller than _5 units__(when we use the same unit for measuring)_
8. Find the image and produce a statement of comparison (bigger than, less than, equal to and so on).
9. What is the sum of all images?__23 units____
10. If you take away the largest bird, what will be the sum the rest of the birds? 19 units___

Image 1. Exercise of measuring and comparing by units.

Another type of exercises consisted in identification of common objects and relations in the sentences and texts of the problems. Image 2 shows this exercise.



2) Pencils, colors, marker, notebook… ___Images_______

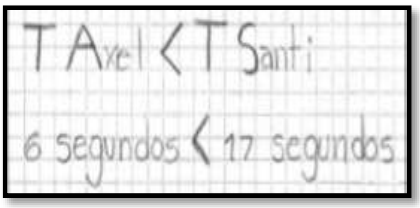
3) 9 units, 3 tens, 1 hundred: __units_______

4) 5 fishes, 3 lunchbox, 2 tomatoes: _quantity of some objects, which have nothing in common___

5) 3 hundred, 2 tens, 1 ten _quantity of the unit of tens____

Image 2. Work with identification of the elements for understanding of the content of the problems.

The exercises included not only the units of longitude, but also units of time. The pupils recorded the seconds needed for crossing the classroom for each of the participants under different conditions and wrote down the results (image 3).

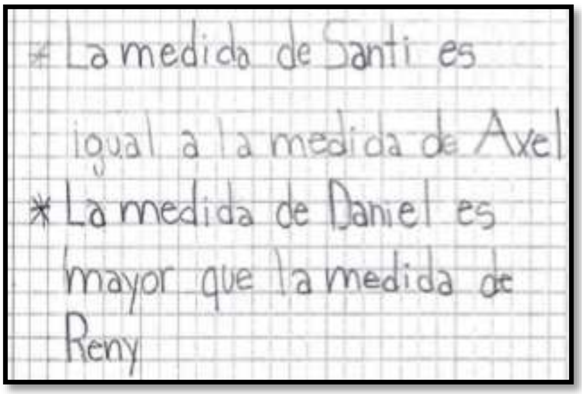


Time Axel < Time Santi

6 seconds < 17 seconds

Image 3. Time records.

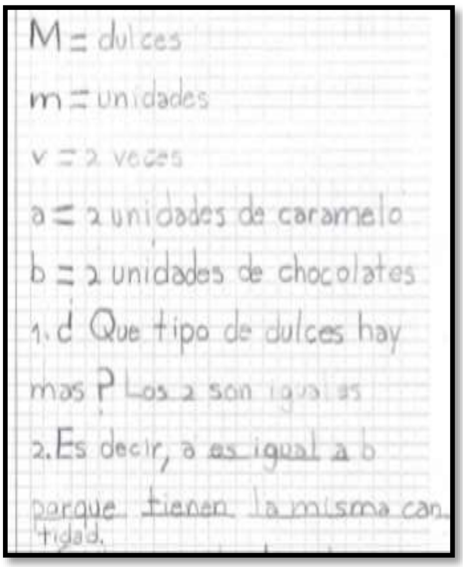
The program included exercises with measuring of volume of liquids. Each pupil used different glasses to fill the huge recipient. Different judgments were obtained about the relation between selected glass, the size and the quantity of measures (image 4).



1) The unit (glass) used by Santiago is equal to the unit for measure used by Axel.
2) The unit for measure used by Daniel is bigger than the unit used by Reny.

Image 4. Judgments of comparison of measurements of volumes.

Different exercises were proposed to the children. The exercises included actions of measuring with different objects and quantities of objects. Specific symbols were proposed for analysis of the components of the action of measure: Unit of measure (m), Magnitude we measure (M), Quantity of repetitions of same unit of measure (v), concrete Images (a, b, etc.). The symbols were external and were written on the blackboard and in the notebooks. Different questions were made in order to establish logic relations between the elements of the action of measuring. The questions were answered in-group using the symbols (image 5).



M= candies
m= unit
v= twice times
a= 2 units of candies
b= 2 units of chocolate

1. Where are more candies? It is the same because we use same unit for measure
2. Which is more: candies or chocolates? It is the same because we use same unit for measure

Image 5. Identification of the components of the action of measure.

Different kinds of tasks were used to work the symbolic operations in external actions. The children had to draw units of measure within the decimal system and in different Images to answer the question: what is more? (Image 6). We can call such exercises as symbolic representation of the decimal system.

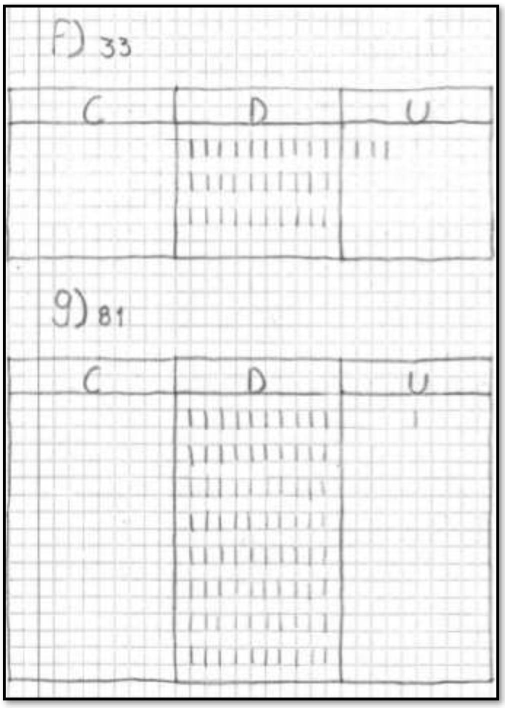


Image 6. Symbolic operations.

Afterwards, the problems were presented according to the level of difficulty (Luria & Tsvetkova, 1981). Firstly, simple problems with only one operation involved were introduced. Later on, complex problems with more than one operation involved were included (table 1).

Simple problem	Complex problem
“Our library “Little Prince” has 40 books divided between 5 shelves. If teacher Lupita puts the same quantity of books on each shelf. How many books should we have on each shelf?”	“Renata and Daniel went to the market and bought 2 kilos of apples, 300 grams of sugar and 1 kilo of pasta. How many grams they have bought altogether?”

Table 1**.** Images of simple and complex problems used during the teaching process.

These Images show that the verbal structure of simple problems is not necessarily also simple. During the work with children, we noticed that the analysis only of numeric data was not enough for reflexive comprehension. We try to show that the word “simple” might be equally used for mathematic content and for verbal content of the text of the problem. It was necessary to let the children notice the difference between “words” used in the text of the problem and numeric data itself. It was possible to do so, working with different problems separating the verbal content (text) and the logic mathematic operation implicated in the problem. Within the content of the program, different verbal content was used to work with the same type of mathematic structure: one operation or more than one operation.

Afterwards, the general orientation for problem-solving was presented to the children. It was explained to them that the problem always mentions a kind of day-to-day ordinary situation. The logic of mathematical solution does not depend on all possible complicity of the words, subjects and objects presented within the text of the problem. The success of the solution of the problem depends on the possibility of substitution of some description by one or more than one arithmetical (mathematical) operations. If it is possible to do so, the problem might be solved. If not, the problem might not be solved in arithmetical terms. Concepts of number and decimal system were essential during this work. Without such concepts it was impossible to solve the problems, or execute arithmetic operations. We noticed that memorization of some data was not specifically useful for the problems. Each problem required an analysis of the content. It was explained to the children that the problem does not contain all data in all occasions. The pupils were explained to search for absent data or to revise the coherence of the content of the problems.

The final objective of the problem is the final question (series of questions) of the problem. In order to find the answer, it is necessary to follow some steps. Children received “orientation card” in order to obtain these steps (image 7). The card was always designed together with the children in the group and the steps were commented by questions and answers reflexively. Each participant had his/her own card, which was used for the solution of each problem. All procedure was planned within the whole group in a collaboration and dialogue. All steps were guided and supervised by the experimenter, the teacher and participants, and discussed collectively.

The work with orientation aimed at forming the reflection of the children for logic intellectual actions required for solution of the problems: reading of the text problem, identification of components of numeric data (M, m, v), solution of numeric operations, verification of the final answer.

Card for the solution of the problems

1.Read the problem and find the question.

2. Look for: M=? m=?, v= ?

3. Find the operation: 1) Ma + Mb = Mc;
2) Ma – Mb = Mc; 3) M = m x v; 4) v = M/m

4. Solve the problem.

5. Have you answer to the question?

Image 7. Orientation card for solution of the problems.

Each point of the "orientation card" implicated work and discussions between all children as follows:

1. Reading the problem aloud and designing of the scheme of the verbal situation expressed in the problem. Identifying the final question and of the elements of the problem represented by symbols (M, m, v). (M= magnitude, objet which is measured (modified); m = the unit of measuring; v= quantity of repetitions of same unit of measure).

2. Designing the plan for solution. Identifying and comparing of relations between data expressed with the words in the problem (M=? m=? v=?)

3. Finding arithmetical operations that corresponded to the operations for the solution. A formula for identification of the unit of measure and election of operations was used. In order to find each operation properly, another card was designed for "Operations for problems" (image 8). Orientation card helped to organize the data between known and unknown for each problem.

4. Fulfilling necessary operation on the blackboard and notebooks.

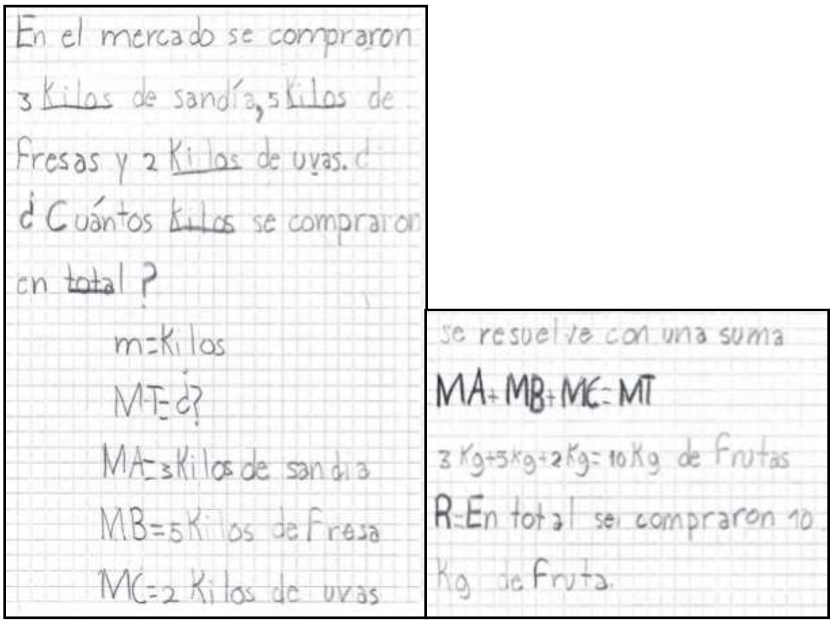
5. Oral reading of the final question and verification of obtained data according to final question.

Arithmetic operations	
If the problem refers an Addition: a) We need to add b) We know the Magnitude c) We know the measure Formula: MA + MB = MC	If the problem refers a Multiplication: a) We need to find Magnitude (M) b) We know the measure (m) c) We know the number of times the measurement was used (v) Formula: M= m × v
If the problem refers a Substracion: a) We need to subtract b) We know the Magnitude c) We know the measure Formula: MA - MB = MC	If the problem refers a Division: a) We need to find the number of times the measurement was used (v) b) We know the measure (m) c) We know the Magnitude (M) Formula: v= M ÷ medida

Image 8. Orientation card for operations.

After the work with the orientation card and its content, different arithmetic problems were presented to the children. The children had to read the problem and to identify all the elements of the problems according to the card. The card was used during all tasks solutions in the group and was discussed orally. The process of reading discussion and solution of the problems was fulfilled as external, guided collective activity. Each child wrote down the elements of orientation and solution.

Image 9 (image 9) shows one of the problems used during the work.



They bought 3 kilos of watermelon, 5 kilos of strawberries and 2 kilos of grapes in the market, how many kilos were bought in total?

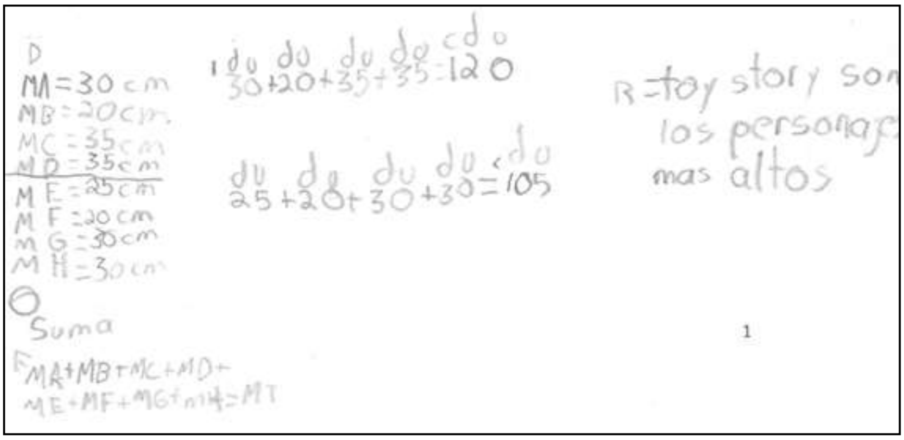
m= kilos
MT= ¿?
MA= 3 kilos of watermelon
MB= 5 kilos of strawberries
MC= 2 kilos of grapes

"I have to use addition"
MA+MB+MC= MT
3kg + 5 kg + 2 kg = 10 kg of fruit

Answer: They bought 10 kilos of fruit.

Image 9. Solution of a simple problem.

In order to analyze the data correctly and reflexively, the problems with lack of data and with abundance of elements were proposed for children. The adults explained that in some cases the problems may not have any solution, if the data presented is not enough. In such cases it would not be possible to answer the final question. In other cases, the text of the problems may mention details that are incidental for the mathematical solution. Such way of reasoning helped to provide a conscious reflection and abstraction of mathematical data from the verbal texts of the problems (Talizina, 2009). Image 10 shows the process of identification of necessary data in the problems about characters of picture Toy Story. The children had to distinguish essential data from irrelevant data (Spiderman). Afterwards, the children wrote the formulas and solved the problem according to the steps.

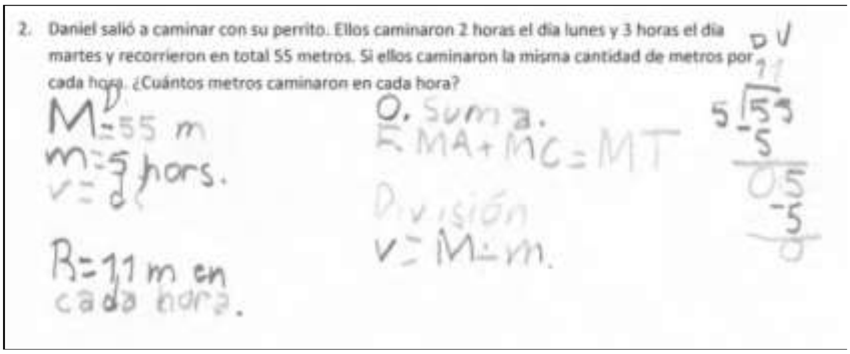


The group of second grade measured the cartoons decorations. The length of the decorations were: Buzz Light-year - 30 cm, Bulls eye - 20 cm, Peter Pan - 25 cm, Jessie - 35 cm, Woody - 35 cm, SpongeBob - 25 cm, Blackboard - 15 cm, Patrick Star - 20 cm, Squid ward Tentacle was 30 cm, Spiderman was 40 cm and Sandy Cheeks was 30

cm. Which character is the highest?

Image 10. Problem with identification of essential data.

Complex problems with more than one operation were included after assimilation of the content of simple problems. Image 11 presents the problem with two operations.



Daniel walked with his little dog. They walked during 2 hours on Monday and 3 hours on Tuesday, they walked 55 meters in total. If they walked the same meters each hour, How meters did they walk in each hour?

Image 11. Problem with two operations.

Image 12 shows the work with conversion of the units of the measure of time (conversion of weeks in days).

Renata has found 42 different puzzles; she wants to do them in two weeks. How many puzzles will she do each day?

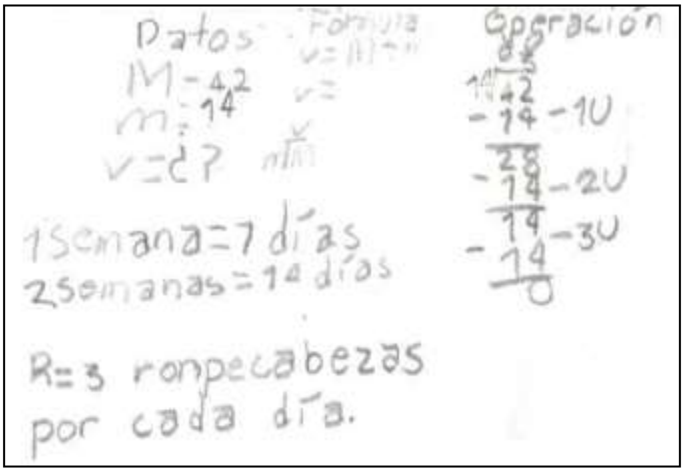


Image 12. Image of the problem with conversion units of time.

After the work with simple and complex problems, we noticed that children were interested to propose their own Images of problems on the basis of the data proposed by the teacher. The work in-group was organized for invention and solution of the problems. The children had to think about 2 aspects: 1) words or verbal text of the problem and 2) numeric data to use in the problem. After invention of the verbal text and numeric data, the process of solution was the same as previously (Images 13 and 14). The children identified the data, used the final question for orientation and solved the problems. Simple and complex problems were invented for each operation: addition, subtraction, division and multiplication.

If Santiago has 50 balls and his father gives him 3, his aunt gives him 25 and his mother 10. How many balls does he have?

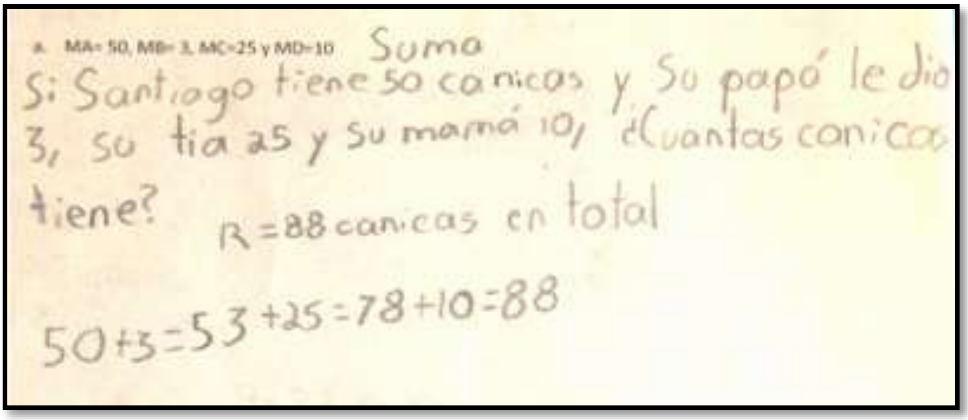


Image 13. Invention of the problem for addition

If Daniel has 40 gums and he gives it to 10 friends, how many gums will be distributed for each boy?

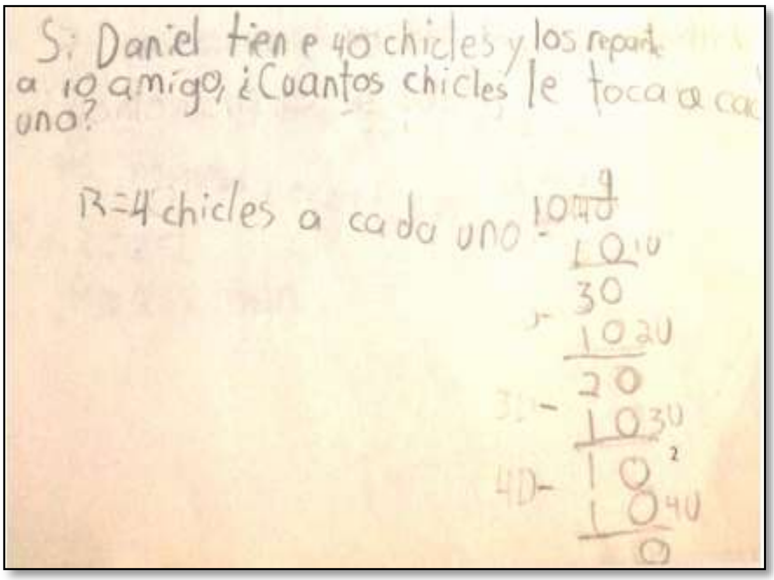


Image 14. Invention of the

problem for division.

Finally, it was clear that the children were ready for totally independent creation and solution of the problems. The children chose the data and proposed different relations between the data and found correspondent word to express such relations. The pupils exchanged created tasks for solution by one of the others pupils in classroom. Image 15 shows the problem created by children.

My plants receive water two times per day. How many times will my plants receive water in 11 days?

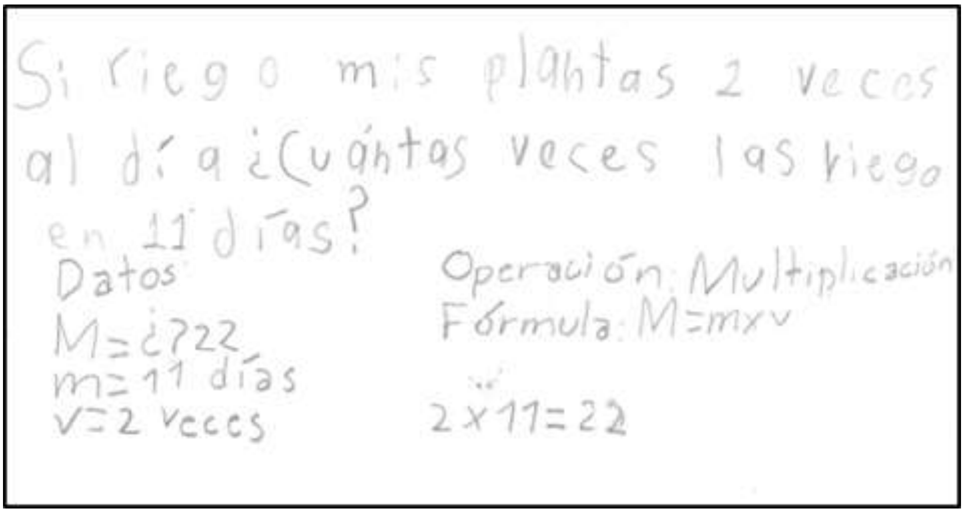


Image 15. Independent creation of the problem by a pupil.

3. Results

The data of initial and final assessment were analyzed according to the types of mistakes committed by children according to the structure of activity of problem solution. Such types of errors were: difficulty to explain the process of solution, impossibility to pass from the verbal content to symbolic operations, difficulties with identification of mathematical concepts and simple guess instead of solution without any reflection (table 2). It was noticed during the initial assessment that the children were unable to solve the problems orally and always needed materialized or perceptive helps.

Table 2.
Types of errors during initial assessment

Structure of activity of problem solution	Types of errors
Identification of the final problem	-Impulsive answers. -Ignoring the final question and direct answer based only on digits of the problem. -Absence of verification of the answer in relation to the

	question.
Identification of the data and election of numeric operation	-Total impossibility to identify relations. -Difficulties to convert data to the one unit of measure. -Difficulties to understand the relation between the verbal text and numeric relations.
The process of solving	-Usage of fingers for operations. -Impulsive operations. -Difficulties to explain chosen operations.

The results of the final assessment have pointed out that the pupils have developed positive logic abilities and assimilated the content of the orientation in the components of the process for problems solving. The solutions were quick, the usage of the formulas was correct and the children were able to solve the problems on verbal level with adequate reflection.

Table 3 shows the answers of one pupil before and after participation in the program as Image of changes in the efficiency of logic operations. The problem presented was: "2 were gone away and 3 birds rested on the tree. How many birds were at the beginning?"

Table 3.
Comparison of the answers of the pupil before and after participation in the program.

Before	After
Pupil: tow Teacher: why so? How do you know? Pupil: it says there were two birds at the beginning. Teacher: do you remember the question? Pupil: yes, how many there were at the beginning and it says there were two. Teacher: what do you have to do to know how many birds there were at the beginning on the tree? Pupil: it says two.	Pupil: there were five birds at the beginning. Teacher: why so? How do you know? Pupil: two birds were gone and three rested on the tree. I have make two plus three, so, we have five birds on the tree, before the two flew away.

The answer of the pupil before his/her participation in the program shows direct usage of the information of the problem without the understanding of the text and numeric relations. The dialog with the teacher does not help to establish relations between the final question and the data and we find only impulsive repetition of the answer, which is wrong. After working with the program, we can observe reflexive and correct understanding and solution of the problem.

Image 16 shows the way of solution of the problem with operation of division during initial and final assessment. Initial assessment shows the drawing of the cake and the parts of it to share between children and usage of numbers 1 and 2. We can see how the child loses the strategy and forgets what 1 and 2 mean in the problem. The strategy used by the child is not useful at all and he gives the answer: 23 cakes. The final assessment shows completely different answer by the same pupil. The child manages to explain the whole procedure and gets correct answer. We can see that initial mistakes have disappeared in the final assessment.

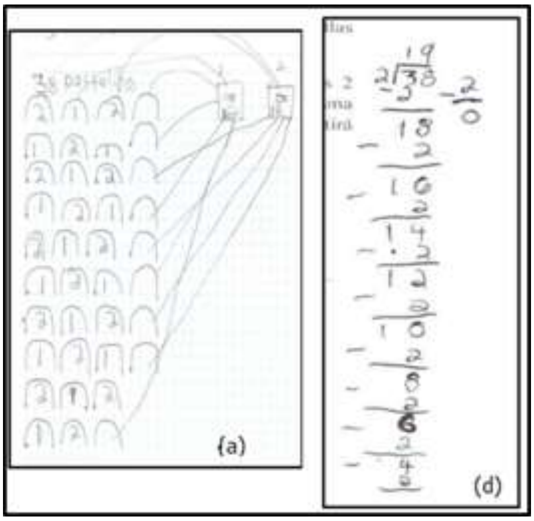


Image 16. Comparison of division of the pupil before and after participation in the program.

The table 4 shows the answers of one of the pupils to the questions about logic relations before and after participation in the program.

Table 4
Task with comparison of measures

Task	Initial evaluation test	Final evaluation test.
Which one is bigger, 3 cm or 1 m?	3, because it is bigger than one.	1 meter is bigger than 3 centimeters, because 1 meter has 100 centimeters
Which one is bigger, 5 liters or 2 kilograms?	5, because it is bigger than two.	Is not possible, they are different measurements
Which one is bigger, two quarters of hour or a half hour?	I don´t know, I haven't been taught to read the clock.	It's equal, because one quarter plus one quarter equals half hour

We can observe that before the participation in the program children always mentioned the biggest number according to the absolute value of the number, without considering the unit of measure. After working with the program, the children started to identify the conversion of units of measures and answered to the questions correctly. The operations of division and multiplication were understood in relation to numeric system and units of measure.

Another important achievement is the appearance and conservation of positive motivation for mathematics in the group. The work with solution and invention of problems became attractive and interesting for pupils, which did not take place before. Before participating in the program, the children had difficulties to understand the purpose of the problems, and proposed unclear Images with huge numbers in order to make it complicated for the other participants. After the work with the program, the children started to invent problems considering objects and situations that were attractive to their mates, and the purpose was always to solve the problem together. We have also noticed that the children started to help and to provide orientation to each other during invention and solution.

4. Discussion

The results obtained in our study show that the acquisition of logic mathematical knowledge and abilities for problems solution may be achieved within the methodology of programed and organized teaching. The program was created on the basis of the activity theory (Leontiev, 1983), and the concept of the zone of proximal development (Vigotsky, 1996). The program considered the structure of the activity of problem solving and the previous logic components (Talizina, 2000, 2009). The usage of symbolic means was introduced starting with external level (Salmina, 1981). Simple problems were followed by complex problems according the structure of operations. The work with the program was organized not as spontaneous individual actions, but as mutual collaboration in classroom between children and adults (Luria, 2006).

We are convinced that the most important part of the program is elaboration and collective usage of specific orientation. Without such orientation the process of solution seems chaotic, a spontaneous process of individual efforts and errors of each pupil. Introduction of orientation permits to achieve positive results during execution without working specifically with execution. As written years ago, we can affirm that the essential part of intellectual activity is orientation, and that execution is nothing without orientation (Galperin, 2002). Our results permit to stress that the essential point of proper orientation for problem solution is the identification of the final question, and the separation of numeric data from the verbal context of the problem.

The inclusion of the components of mathematic thinking (symbolic, logic and numeric actions) according to the activity theory guarantees to understand and assimilate reflectively the mathematical knowledge, as it was shown in previous studies (Rosas, Solovieva & Quintanar, 2014; Rosas & Rosas, 2011; Zárraga, Solovieva & Quintanar 2012; Solovieva, Ortiz & Quintanar, 2010; Salmina, 2001; Nikola & Talizina, 2001). Consideration of these components permits to continue the progressive acquisition of the content of mathematical knowledge at primary school. The children begin to separate reflectively absolute and relative values of numbers and positional symbolic meaning of digits. Some authors have described the absence of such abilities as the most frequent difficulties during learning of mathematics at school. The authors stress that symbolic components are frequently not considered by children and they read number 329 as divided in components 300-20-9 and do not manage to integrate them into a complex numeric system. Such situation provided confusion and logic mistakes in pupils (Castaño, 2008; García & Rodríguez, 2009).

According to the literature, the process of problems solving is based not only on arithmetic operations, but also in the comprehension of basic elements of the palled situation and relation between them (Vicente, Dooren & Verschaffel, 2008).

The study of Silva and Rodríguez (2011) have detected severe difficulties in pupils of the sixth grade of primary school with the following parts of mathematical knowledge: 1) absence of previous basic knowledge, 2) misunderstanding of problems, 3) luck of strategy of planning, 4) difficulties with execution of operations and 5) impossibility to verify results. Other studies have related difficulties in the process of teaching of mathematics to 1) type of strategy to understand the problem, 2) necessary conceptual knowledge and 3) semantic variances of the problem (Orrantia, González & Vicente, 2005). This last point is especially important, because we agree that the separation of the semantics of the verbal text of the problem and numeric and logic relations are essential parts to achieve the understanding of the problem. The verbal part of the problem is one on the components of problem solving as intellectual verbal activity (Luria y Tsvetkova, 1981; Talizina, 2001).

Previous research (Martínez, 1984; Nikola & Talizina, 2001) has detected that the teaching of problems at school should not be reduced to a mechanic solution, but includes reflective analysis of the components of the problem. We absolutely agree with this opinion and we add the necessity of separation of the verbal and numeric aspects of the problem as a reflective action of the children. One of the aspects included in orientation in our program was precisely work with the text and logic relations.

It is important to mention that actions with mathematical knowledge at primary school might not be reduced to operations of addition, subtraction, division and multiplication. These operations might be understood reflectively by means of inclusion in the content of mathematical problems with different structure. According to Talizina (2001), Tsvetkova (1999) and Luria (2006) the problems should be taken into account as logic and intellectual activity. Traditional school education does not consider the importance of gradual formation of intellectual activity and insists only on technical repetition and memorization of operations. Specific orientation and work with components of intellectual activity permits to introduce the work with solution of problems in classroom. Important previous concepts are the concepts of number and decimal system. Important symbolic and logic actions are the identification of unit of measure and times of application of units during measuring of different magnitudes. Reflective introduction of symbolic external means for measure and formulas for actions is an important part of elaboration of orientation. Such orientation might be called as complete, generalized and independently reflective (Galperin, 2009a).

The work with the program according to the method of gradual formation of mental actions (Galperin, 2009b, c; Talizina, 2001, 2009) has permitted to divide the content of the problems solving process into essential components and to present each component as an object of actions (Leontiev, 1983). According to the activity theory, each object of intellectual action should firstly be presented as an external object, and later children might interiorize this object. This methodology is opposite to proposals based on direct observation of strategies used by children spontaneously, and later the introduction of useful strategies (Butto & Gómez, 2011). Other proposals are based on the idea of personalization of the teaching process by inclusion of previous knowledge and social context insisting in pragmatic learning or cognitive stiles (Toledo, Pérez, Riquelme, Hernández & Bittner, 2011). The idea of Gaplerin of the "teaching without mistakes" implies the usage of correct strategies at the very beginning of the process based on argued and fundamental orientation.

In opposition to these proposals, we considered that the only way for proper acquisition of knowledge at school and assimilation of mathematic concepts is the organized collective work based on previously elaborated orientation. Such organization include the gradual interiorization of intellectual actions by stages: material, perceptive, external verbal and internal verbal (Galperin, 2009a, Nikola & Talizina, 2001; Cervantes, 2009; Solovieva, Ortiz & Quintanar, 2010; Solovieva & Quintanar, 2010; Álvarez & Del Río, 2013).

The work on the basis of the zone of proximal development, according to Vigotsky's conception, does not mean the presentation of something, what the child knows form the context. The work in the zone of proximal development means introduction of new knowledge, which is accessible for the child in situation of collective collaboration. This is Vigotsky's true proposal (1991, 1996). And this was done by the implementation of the proposed program.

While working with the program, the children became convinced that they were capable students and they could understand, resolve and even create new problems by themselves. The teaching process was represented not as individual internal process of each child, but as a joint collective activity. The participants at all moments of the work shared their cognitive experiences and emotional involvement (Del Río & Álvarez, 2011). We can affirm Davidov's position that the content of educational process and teaching determine the psychological development of schoolchildren (Davidov, 1988).

6. References

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